COMUE L'UNAM
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Spécialité : Mathématiques
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Équipe : Algèbre et Géométrie

Plane curves and the symmetry of values

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Parametrization

Let $C = C_1 \cup \cdots \cup C_p \subset \mathbb{C}^2$ be a plane curve germ defined by a reduced equation $f(x,y) \in \mathbb{C}\{x,y\}$, with parametrization $(\varphi_1(t_1), \ldots, \varphi_p(t_p))$ and ring $\mathscr{O}_C = \mathbb{C}\{x,y\}/(f(x,y))$.

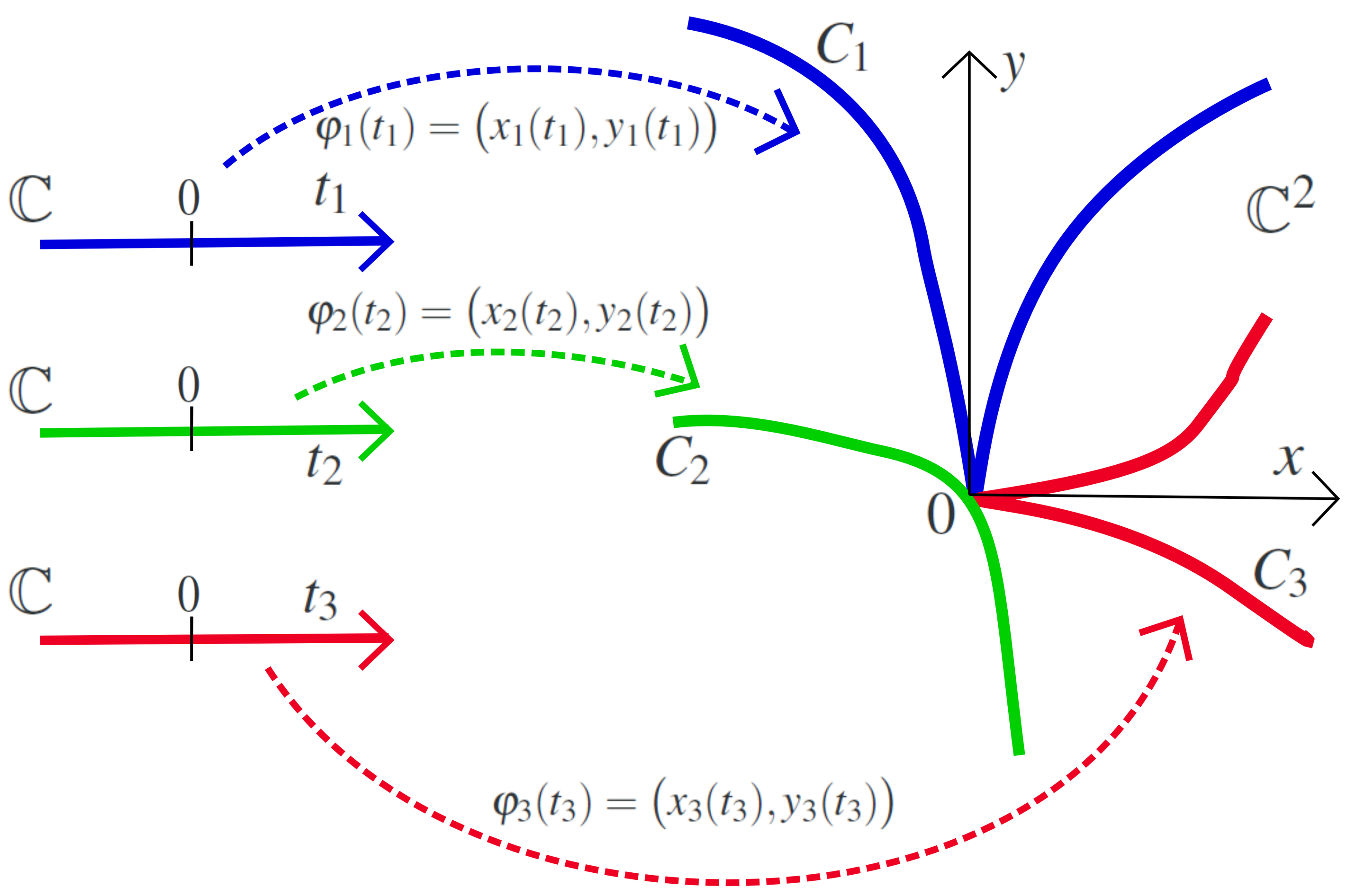


Figure : Parametrization of a reducible plane curve

Initial statement

Definition : Value and conductor

•Let $g \in \mathrm{Frac}(\mathscr{O}_C)$ be a fraction. We define :

$$\mathrm{val}_i(g) = \text{lowest power of } t_i \text{ in } g(\varphi_i(t_i))$$

$$\mathrm{val}(g) = (\mathrm{val}_1(g), \ldots, \mathrm{val}_p(g)) \in \mathbb{Z}^p$$

•The conductor is the lowest $\gamma \in \mathbb{N}^p$ such that

$$\gamma + \mathbb{N}^p \subseteq \mathrm{val}(\mathscr{O}_C)$$

Definition : Dual of an ideal $I \subseteq \mathscr{O}_C$

The dual of I is : $I^\vee := \{m \in \mathrm{Frac}(\mathscr{O}_C) \ ; \ mI \subseteq \mathscr{O}_C\}$.

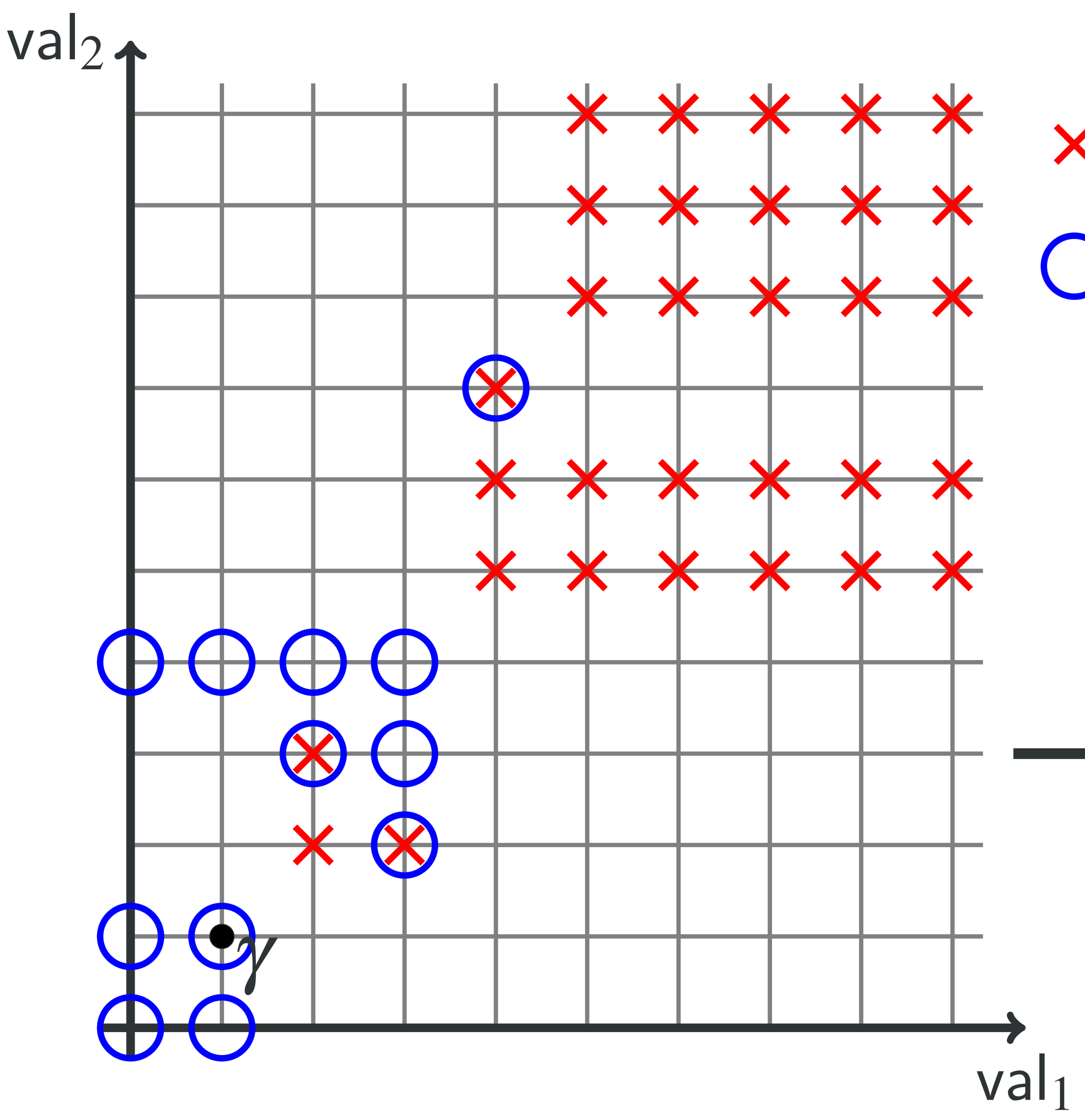
<u>Proposition</u> : If C is irreducible $(p = 1)$,

$$v \in \mathrm{val}(\mathscr{O}_C) \iff \gamma - v - 1 \notin \mathrm{val}(\mathscr{O}_C)$$

Question

Is there an analogous relation between the values of an ideal and the values of its dual ?

The statement for two branches $(p = 2)$



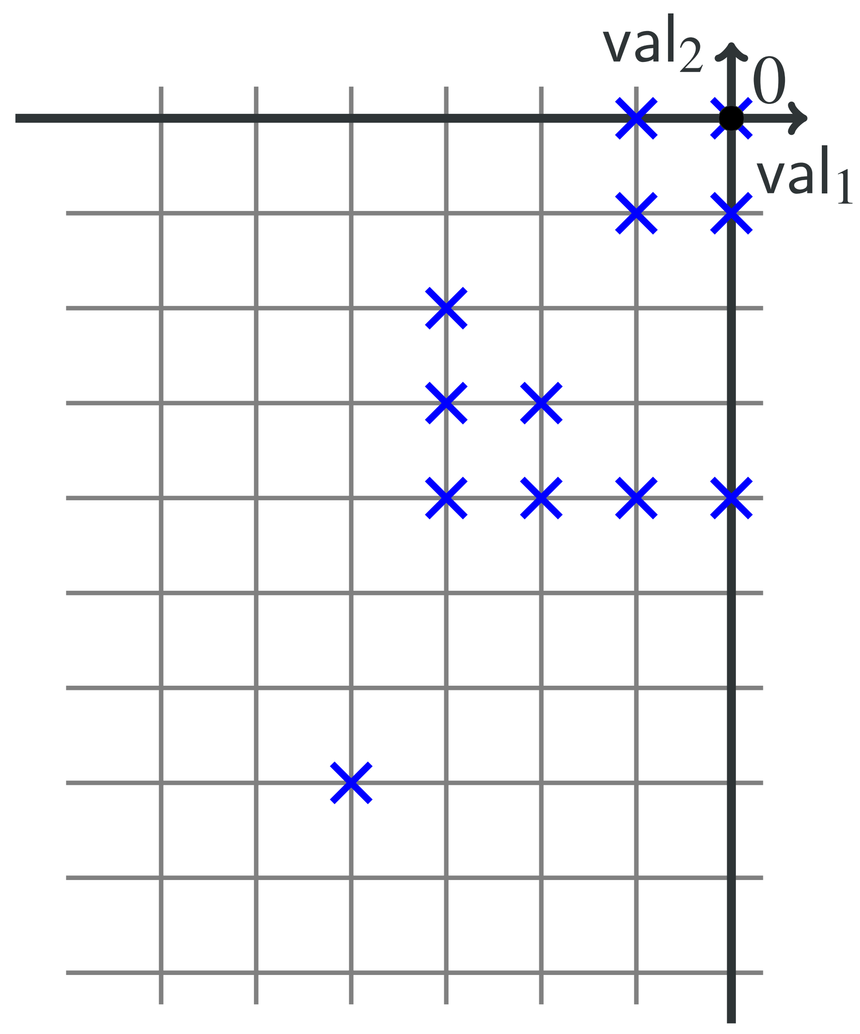
×: $\mathrm{val}(I)$ ×: $\mathrm{val}(I^\vee)$

○: no × above nor on the right

Theorem ([3], 2.4):

○ gives ×

by $\gamma - v - (1,1)$



Conclusion and perspectives

- In fact, there is a symmetry property for all $p \geqslant 1$. The proof is based on a computation of dimensions from the values.
- This result is useful to study the behaviour of the Jacobian ideal and its dual, the logarithmic residues in deformations of curves.
- It implies also an interesting relation between logarithmic residues and the problem of analytic classification of plane curves.

References

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[2] M.Granger, M.Schulze, Normal crossing properties of complex hypersurfaces via logarithmic residues, Compos. Math., 150(9), 1607-1622, 2014.

[3] D.Pol, Logarithmic residues along plane curves, C. R. Acad. Sci. Paris, Ser. I, 353(4), 345-349, 2015.